

As you move on into advanced math, you'll see lots of problems like the one at right. One of the most difficult things in math, for many people, is reading an abstract problem like this and understanding it, making sense of the math behind it.

Make no mistake—this is a tough math problem. But you have most of the math you need to solve it now. Put it aside for the moment, we'll come back to it tomorrow.

**Abstract Representation:**

Given an arbitrary point  $P$  on a line segment  $AB$ , let  $AP$  form the perimeter of a square and  $PB$  form the circumference of a circle.

Find  $P$  such that the area of the square and circle are equal.

One of the things we high school math teachers do is make problems more "accessible:

The problem at right is almost entirely identical to the abstract representation above. But now it has concrete images and a situation to be visualized.

**Concrete Representation:**

Wu Zetian, the only female emperor in Chinese history, had two pandas, Sing and Ping. These greatly cherished pets had to be treated equally, while still granting the placid animals their own idiosyncrasies. Sing, for example, found total zen in square environments, while Ping harmonized with circular ones. The queen wanted sleeping pens for her beloved bears, one in a perfect circle, the other a perfect square.

She had just been given a gift of 100 meters of gold thread webbing all in one piece, beautiful material she wanted to use as fencing for the pens, wrapped around fencing poles. Naturally, Sing and Ping must have equal sleeping areas, and not a shred of the gold webbing must be wasted!

Where should the webbing be cut so that each piece will enclose a sleeping pen of equal area for Ping and Sing? The carpenters can only cut once, so their lives depend on your answer!

1. Underline the key question in the problem.



2. The adorable panda images aren't just there to look pretty. Once you've solved the problem, the carpenters will be using two pieces of webbing to build....what? Outline or trace the relevant part of the panda pictures.

3. Each piece of webbing will ENCLOSE a pen—one that is square, one that is a circle. Cast your mind way, way back to geometry, and see if this sounds familiar. Two formulas should come to mind.

a. \_\_\_\_\_

b. \_\_\_\_\_

4. And then, two other formulas probably come to mind, mentioned in the problem directly:

a. \_\_\_\_\_

b. \_\_\_\_\_



Now you have much of the information you need to start the problem.

## Part I

Then we math teachers can make it even more concrete by giving you a manipulative.



Gold webbing! Kind of.

In class, you've been working with systems: two or more equations that use the same variables so they can be combined to find specific solutions. This problem represents a system of two equations. But before you can solve the system, you need to identify and model the equations.

Your gold webbing will help you think through the problems involved. You should have a chain of 100, with colored clips marking each 20.

System Variables:

P: The part of the webbing used to make the square

C: The part of the webbing used to make the circle

You can work this problem any way you like, but at the end of this section you will have identified the following:

1.  $P + C =$  \_\_\_\_\_
2. What is the area of the square, in terms of P? \_\_\_\_\_
3. What is the area of the circle, in terms of C? \_\_\_\_\_

### Suggestions:

Break the chain up into two pieces. I suggest pieces that are multiples of 4. (WHY? Important thing to consider).

Form the pens. What's the area? How did you find it? What about the circle?

Notice IN TERMS OF. This is not a case of guess and check.

## Part II

Another area we've been working with is transformations. Each of these graphs is a transformation of a basic parent function. Get out your transformation handout notes from last week and, with your group, identify the type of transformation made to each graph from their parent function.

Equations again

Parent function

Transformation:

- |                          |       |       |
|--------------------------|-------|-------|
| 1. $P + C =$ _____       | _____ | _____ |
| 2. Area of Square: _____ | _____ | _____ |
| 3. Area of Circle: _____ | _____ | _____ |

## Part III

Now you're ready to define and solve the system. Remember, in all three equations, P and C will be equal.

**Use P as your primary variable.**

Will combination work in this system? (say NO!) Which leaves what method? \_\_\_\_\_

If you look at your work in Part II, you'll see that you've created the equation already.

Plug it in, and off you go. At a certain point yes, you can use a calculator. Put your solution here—take a picture of your boardwork when done and put it here—you want it for later! This is a good worked example of some tough algebra:

Part IV

Now that you've found the answer, consider these questions:

1. First, what is the answer? Go back to the first page, check the underlined section. Put the specific answer to that question here:
2. Having found  $P$ , what percentage of the 100 yards does it represent? \_\_\_\_\_
3. What if the webbing was only 60 yards? Look at your original equation—it will be pretty easy to change and run again, now that you know how. Once you find the answer, find out what percentage it is of the total 60 yards. Compare it to the answer in problem 2. Using this information, can you determine what the correct cut would be if the webbing were 180 yards?
4. What shape encloses area more efficiently---a square or a circle?
5. The carpenters went back to the Empress, groveling, and told her that, while they were happy to accede to her wishes, they wanted her to know that Ping and Sing would have much more room to sleep if they never cut the webbing. Is this true? Which bear will be happier with the results, if the Empress doesn't disembowel the carpenters for their audacity?

#### Part V--Challenge

Remember the abstraction?

**Given an arbitrary point  $P$  on a line segment  $AB$ , let  $AP$  form the perimeter of a square and  $PB$  form the circumference of a circle.**

**Find  $P$  such that the area of the square and circle are equal.**

Create a sketch and use your work from the previous section to find  $AP$  in terms of  $AB$ .